

Hellenistic Technology, Mathematic, and Astronomy

Alena Šolcová

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Aristarchus of Samos, ca. 310-230 BC

- He was very knowledgeable in all sciences, especially astronomy and mathematics.
 - He discovered an improved sundial, with a concave hemispherical circle.
 - He was the first to formulate the Copernican hypotheses and is sometimes called the **Ancient Copernican**.
 - He countered the *nonparallax* objection by asserting that the stars to be so far distant that parallax was not measurable.
 - Wrote ***On the Sizes and Distances of the Sun and Moon***.
 - In it he observed that when the moon is half full, the angle between the lines of sight to the sun and the moon is less than a right angle by $1/30$ of a quadrant.
 - From this he concluded that the distance from the earth to the sun is more than 18 but less than 20 times the distance from the earth to the moon. (Actual).
- He also made other trigonometric estimates -- without trigonometry.

A Man of Golden Age - Apollonius of Perga

- **Apollonius** was 25 years younger than **Archimedes**.
- They together with **Euclid** stood well above all other mathematicians of the first century of this period.
- Because of them, this period is sometimes called the “**golden age**” of Greek mathematics.

Apollonius of Perga, ca 262 BC-190 BC

- Apollonius was born in Perga in Pamphilia (now Turkey), but was possibly educated in Alexandria where he spent some time teaching.
- Very little is known of his life.
- He seems to have felt himself a rival of Archimedes.
- In any event he worked on similar problems.
- He was known as the “great geometer” because of his work on conics.

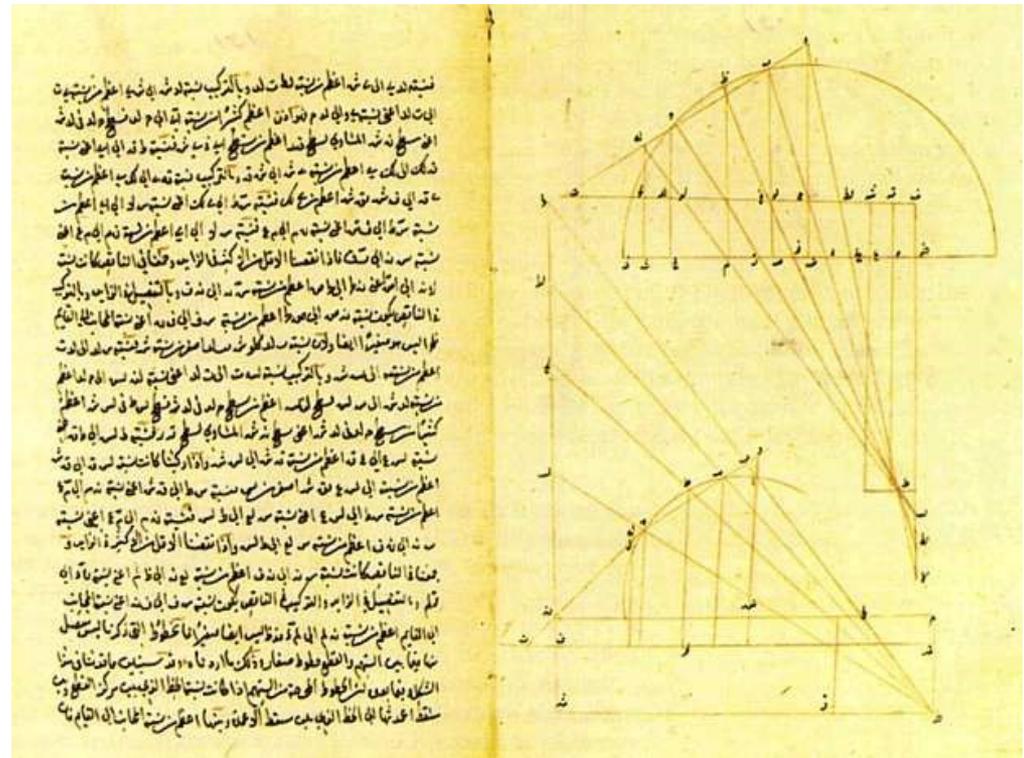
Apollonius' papers

Apollonius wrote many books.

All but one are lost.

Among those we know he wrote are:

- *Quick Delivery*
- *Cutting-off of a Ratio*
- *Cutting-off of an Area*
- *On Determinate Section*
- *Tangencies*
- *Vergings (Inclinations)*
- *Plane Loci*



Apollonius of Perga II

- In his book **Quick Delivery** (lost), he gives the approximation to π as 3.1416. We do not know his method.
- His only known work is ***On Conics*** - 8 Books, only 4 survive.

Features:

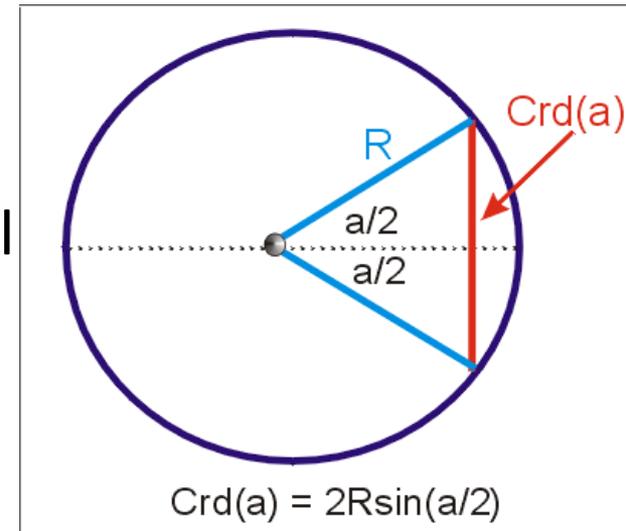
- *Using the double oblique cone he constructs the conics parabola, ellipse, hyperbola, whose names he fixed for all time.*
- He made use of the idea of **Symptoms** which were similar to equations, there results **an analytic-like geometry - but without coordinates!**
- **Proposition I-33.**
- If AC is constructed, where $|AE| = |ED|$, then AC is tangent to the parabola.

Apollonius of Perga III

- He improved upon the numbering system of Archimedes by using the base 10^4
- In the works of Apollonius,
Greek mathematics reached its zenith.
- Without his predecessors, the foremost being Euclid, Apollonius never would have reached this height. Together, they dominated geometry for two thousand years.
- With the works of Apollonius, mathematics was now well beyond the reaches of the dedicated amateur. Only a professional would be able to advance this theory further.

Hipparchus (fl 140 B.C.)

- **Hipparchus of Nicaea** was a scientist of the first rank.
- So carefully accurate were his observations and calculations that he is known in antiquity as the “**lover of truth.**” He worked in nearly every field of astronomy and his reckonings were canon for 17 centuries!
- Only one work of his remains, a commentary on *Phainomena* of Eudoxus and Aratus of Soli. We know him, however, from Ptolemy's *The Almagest*.
- **Indeed the “Ptolemaic Theory” should be called Hipparchian.**
- His mathematical studies of astronomical a computation of a table of sines. He constructed a table of chords for astronomical use.



Hipparchus II

- Hipparchus knew the half angle formula as well.
- He could compute the chord of every angle from 0 to 180
- Using no doubt Babylonian observations, he improved the lunar, solar and sidereal years. He reckoned the solar year at 365 days, less 4 minutes, 48 seconds -- an error of 6 minutes from current calculations.
- He computed the lunar month at 29 days, 12 hours, 44 minutes, 2 seconds -- less than one second off.
- He also computed the synodic periods of the planets with astonishing accuracy.
- He estimated the earth-moon distance at 250,000 miles, less than 5 percent off.
- Hipparchus almost concluded the orbit of the earth about the sun to be elliptic through his theory of “eccentrics” to account for orbital irregularities.



Syene, Alexandria

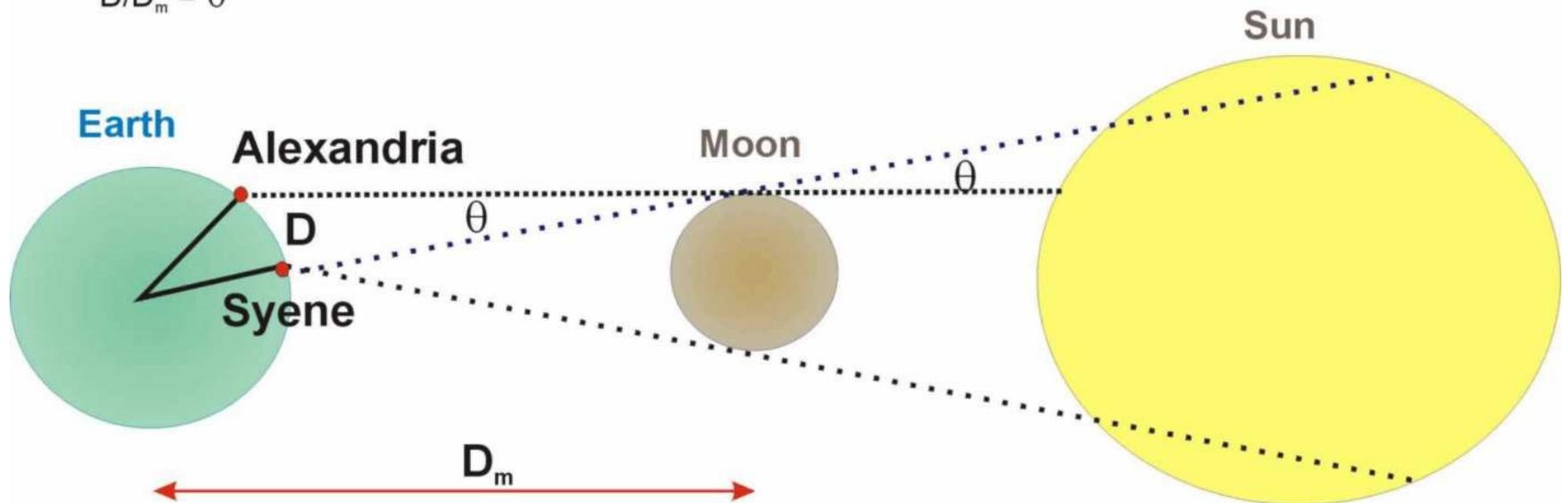
Line of sight from Alexandria

Area of shadow produced from the moon

When a total eclipse of the sun is observed in Syene, in Alexandria an observer saw 1/5 of the sun

Then angular size of the moon and the sun is $1/2^\circ$, $\theta = 1/10^\circ$

$$D/D_m = \theta$$

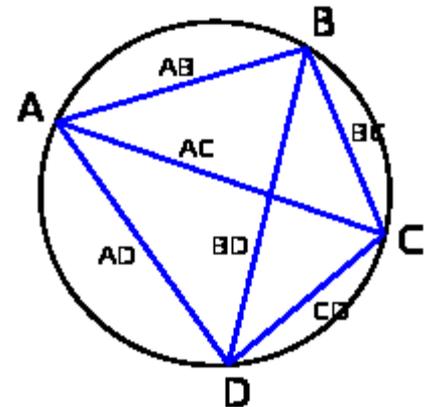


Hipparchus III

- In about 129 B.C., he made a **catalogue of 1080 known fixed stars** in terms of celestial longitude and latitude.
- Comparison of his chart with that of **Timochares** from 166 years earlier, he made his most brilliant observation.
- Noting a shift in the apparent position of the stars he predicted the precession of the equinoxes -- the advance, day by day, of the moment when the equinoctial points come to the meridian.
- He calculated the precession to be 36 seconds/year -- 14 seconds slower than the current estimate of 50 seconds.

Claudius Ptolemy, ca. 100 – ca. 150 AD

- Ptolemy was an astronomer/mathematician
- He wrote
 - *The Geography*-- a compilation of places in the known world along with their geographic coordinates.
 - *Mathematical Collection*, later called **The Almagest**, for the Arabic ,al-magisti'.
- **Mathematical Collection** - 13 books, which contains
 - table of chords - 180 intervals -- $R=60$, in sexagesimal.
 - He solve triangles, planes and spherical triangles.
 - The book was very algorithmic - this was a text/tutorial not a research monograph.
 - We also find the well known **Ptolemy's Theorem**:
$$AC \cdot BD = AB \cdot CD + AD \cdot BC$$
- *The Almagest* was the standard of astronomers until the time of Copernicus.



Heron of Alexandria

1 century A.D.

- Very little is known of Heron's life. However we do have his book: ***Metrica*** which was more of a handbook. In it we find the famous **Heron's formula**.

For any triangle of sides a , b and c , and with semiperimeter $s = a + b + c$, the area is given by

$$A = \sqrt{[s (s - a) (s - b) (s - c)]}$$

- He also gives formulas for the area of regular polygons of n sides, each of length a :

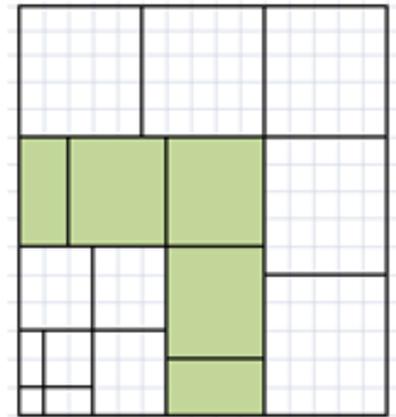
Nicomachus of Gerasa, fl.100 AD

- Nicomachus was probably a neo-Pythagorean as he wrote on numbers and music.
- The period from 30BC to 641 AD is sometimes called the **Second Alexandrian School**.
- He studied in Alexandria.
- Only two of his books are extant:
 - ***Introduction to Arithmetic***
 - ***Introduction to Harmonics***
- A third book on geometry is lost.
- **Introduction to Arithmetic** - in two books
- Book I is a classification of integers
- and their properties without **proofs**.

Nicomachus of Gerasa II

- Classification of ratios of numbers.
Assume a/b is completely reduced form of A/B (i.e. a and b are relative prime).
- Book II discusses plane and solid numbers but with .
(Were the proofs removed in the many translations?)
He studies
the very Pythagorean:
triangular numbers /
square numbers /
pentagonal numbers /
hexagonal numbers /
heptagonal numbers, etc.

$$\left(\sum_{n=1}^k n\right)^2 = \sum_{n=1}^k n^3$$



Nicomachus of Gerasa III

- He notes an interesting result about cubes:
- This should be compared with the summation of odd numbers to achieve squares.
(Recall, the square numbers of Pythagoras.)
- The other known work, *Introduction to Arithmetic* was a “handbook” designed for students, primarily.
It was written at a much lower level than Euclid's *Elements* but was studied intensively in Europe and the Arabic World throughout the early Middle Ages.

Other Great Geometers

- **Hypsicles of Alexandria** (fl. 175 BC)
- **Diocles of Carystus**, (fl. 180 BC)
- **Nicomedes**, (fl. 260 BC)
- **Eratosthenes of Cyrene**, (c. 276 - c. 195 BC)

Hypsicles of Alexandria (fl. 175 BC)

- Added a fourteenth book to the *Elements* on regular solids.
- In short, it concerns the comparison of the volumes of the icosahedron and the dodecahedron inscribed in the same sphere.
- In another work, *Risings*, we find for the first time in Greek mathematics the right angle divided in Babylonian manner into 90 degrees.
- He does not use exact trigonometry calculations, but only a rough approximation.
For example, he uses as data the times of rising for the signs from Aries to Virgo as an arithmetic progression.
- He also studied the **polygonal numbers**.

Diocles of Carystus, fl 180 BC

- **Diocles of Carystus**, fl 180 BC, invented the **cissoïd**, or ivy shaped curve. It was used for the duplication problem.
- Diocles also solved an *open problem* posed by Archimedes, that of dividing a sphere into two parts whose volumes have a prescribed ratio.
- He was able to solve certain cubics by intersecting an ellipse and a hyperbola.
- He studied refraction and reflection in the book ***On Burning Mirrors***.

Nicomedes, (fl. 260 BC)

- **Nicomedes**, (fl. 260 BC),
wrote *Introduction Arithmetica*.

He discovered the **conchoid** and used it for angle trisection and finding two mean proportionals.

Eratosthenes of Cyrene, c. 276 - c. 195 BC

- Eratosthenes achieved distinction in many fields. and ranked second only to the best in each. His admirers call him the second Plato and some called him *Beta*, indicating that he was the second of the wise men of antiquity. By the age of 40, his distinction was so great that Ptolemy III made him head of the Alexandrian Library.
- He wrote a volume of verse and a history of comedy.
- He wrote mathematical monographs and devised mechanical means of finding mean proportions in continued proportion between two straight lines. He also invented the **sieve** for determining primes.

Eratosthenes II

- In a remarkable achievement he attempted the measurement of the earth's circumference, and hence diameter.
- Using a deep well in Syene (nowadays Aswan) and an Obelisk in Alexandria, he measured the angle cast by the sun at noonday in midsummer at both places.
- He measured the sun to be vertical in Syene and making an angle equal to $1/50$ of a circle at Alexandria. With this data he measured the circumference of the earth to be 25,000 miles.
- Remember, this measurement of the radius of the earth was made in 250 B.C. The diagram -> see next page:

Eratosthenes III

