

Computing before Computers

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March 15, 2018

Sketch of Leonardo da Vinci

Manuscripts

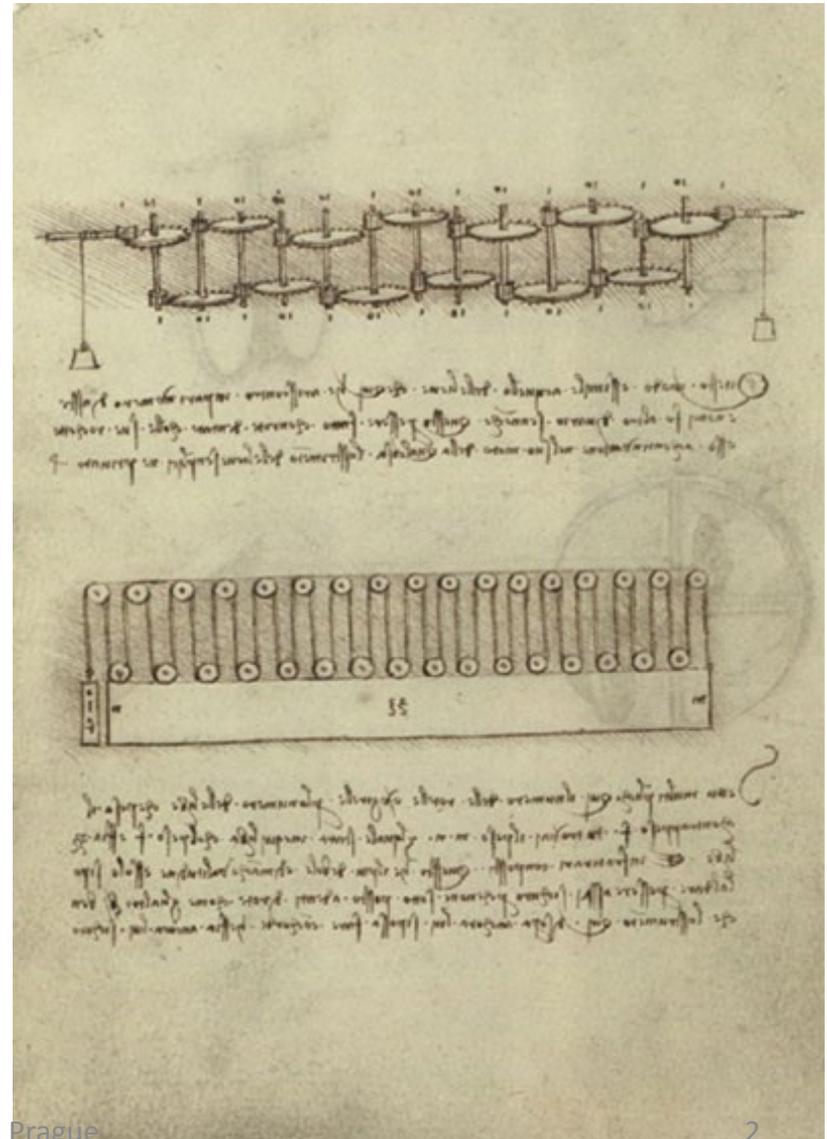
almost 700 pages,
on subjects such as

- architecture,
- geometry,
- music,
- mechanics,
- navigation and maps

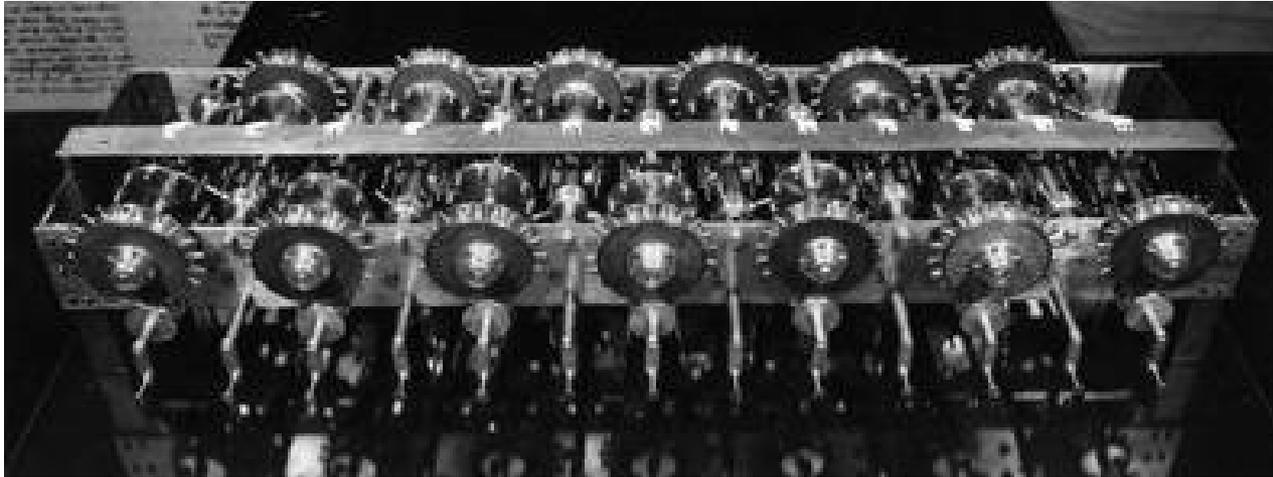
are now referred to as

Madrid Manuscripts or
Codex Madrid I and
Codex Madrid II.

The gear wheels in the figure are numerated as follows: **the small wheels are numerated with 1,** **while the bigger wheels are numerated with 10** (take into account, that in this case Leonardo, as in many of his writings, writes laterally inverted from right to left!).



Leonardo's Device for Calculation- Replica



- An early version of today's complicated calculator, Leonardo's mechanism maintains **a constant ratio of ten to one in each of its 13 digit-registering wheels.**
- For each complete revolution of the first handle, the unit wheel is turned slightly to register a new digit ranging from zero to nine. Consistent with the ten to one ratio, the tenth revolution of the first handle causes the unit wheel to complete its first revolution and register zero, which in turn drives the decimal wheel from zero to one.
- **Each additional wheel marking hundreds, thousands, etc., operates on the same ratio.** Slight refinements were made on Leonardo's original sketch to give the viewer a clearer picture of how each of the 13 wheels can be independently operated and yet maintain the ten to one ratio.
- Leonardo's sketch shows weights to demonstrate the equability of the machine.

Conclusion of Leonardo

- *Leonardo was possibly studying the properties of gear trains in comparison with systems of levers; both can multiply forces (torques), but only gears can produce a continuous movement. In the other direction the gear train can multiply rotation speed.*
- *In the same page, in fact, a compound pulley system is shown, which has the same force-multiplying properties as a gear train, a demonstration of what Leonardo was examining.*
- *I can only add some points:*
 1. *Leonardo's drawing does not show any numbering on the gear wheels (mandatory for a calculator).*
 2. *No way to set the operands is shown (which is mandatory).*
 3. *No way (e.g. ratchet) to stop the wheels in precise discrete positions (which is mandatory) is shown.*
 4. *Two weights are shown at the two ends (useless in a calculator).*
 5. *The use of 13 decimal figures for calculations in XV century is quite a question.*
- **Even if the mechanism of Leonardo was designed for calculating purposes, it was probably not built and had no influence over the further development of the mechanical calculating devices.**

The *Calculating Clock* of Wilhelm Schickard



- Wilhelm Schickard was born in the small south German town Herrenberg, near Tübingen, and educated in the Protestant theological seminary Tübinger Stift, in Tübingen.
- He received his bachelor degree in 1609 and master degree in theology in 1611. In 1613 he became a Lutheran minister, continuing his work with the church until 1619, when he was appointed professor of Hebrew at the University of Tübingen.
- In 1631 he became a professor of astronomy, mathematics and geodesy.

Meeting with Johannes Kepler

- Undoubtedly **one of the most important events in the life** of the modest deacon was his meeting in October, 1617, with the great astronomer **Johannes Kepler**.
- Kepler, just like Schickard, had studied theology at Tübinger Stift. Kepler visited Tübingen during one of his journeys in Württemberg, to see his old friend **Michael Maestlin** (also Mästlin, Möstlin, or Moestlin, (1550-1631), a famous German astronomer and mathematician, who used to be a mentor of Johann Kepler) and others.



Maestlin probably was some kind of a patron for Schickard (as he used to be for Kepler), because at that time there was no academic appointment without patronage.

Schickard –an excellent talent, a math loving young man



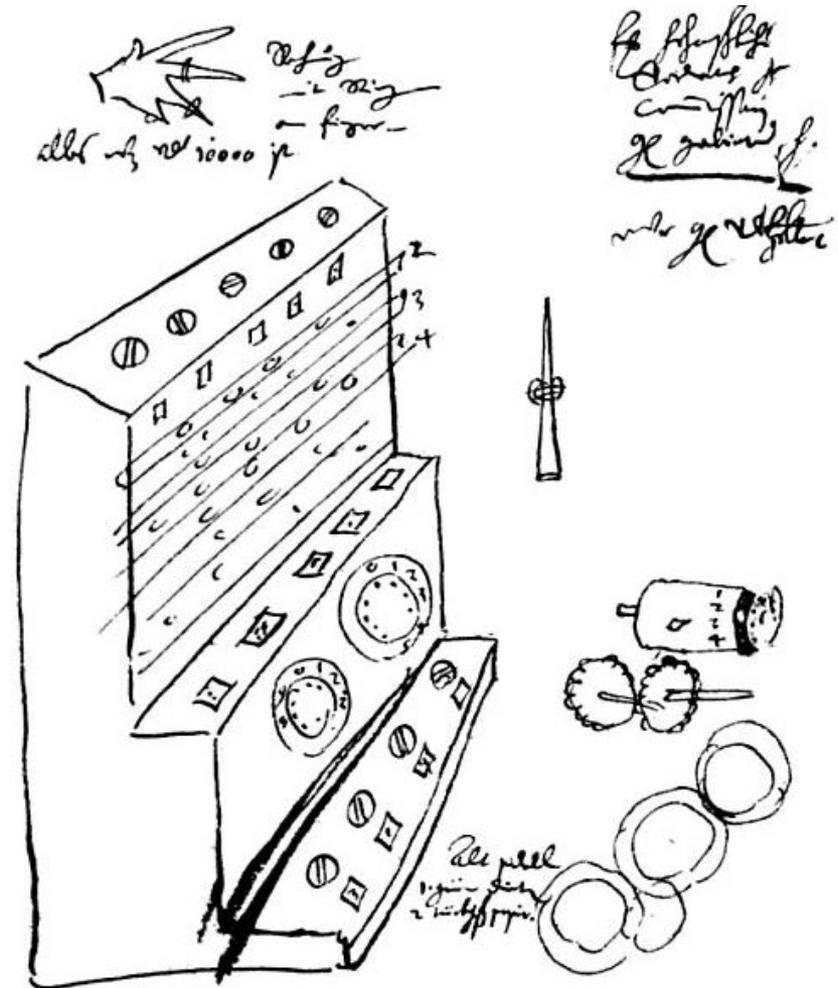
- Kepler wrote in his diary for his first impressions of Schickard
"In Nürtingen I met also an excellent talent, a math-loving young man, Wilhelm, a very industrious mechanic and lover of oriental languages."
Obviously during this meeting Kepler immediately recognized the massive intellect of the young Wilhelm, and encouraged his occasions with sciences.
- From this moment on, Schickard entered into close friendship and busy correspondence with Kepler until his death, made science investigations for him, **took care for Kepler's son—Ludwig**, who was a student in Tübingen, created by Kepler's request figures and copperplates, and helped for the printing of Kepler's renown books, and which is most interesting for us—designed a mechanical calculating machine
Schickard referred to it as *Rechen Uhr—calculating meter or calculating clock*, which proved to be the first mechanical calculating device ever created.

Letters in Pulkovo

- Unfortunately, the machine, designed by Schickard around 1623, didn't manage to survive to the present day. Only 3 documents about this machine have been found till now- **two letters from Schickard to Kepler, and a sketch of the machine with instructions to the mechanician.**
- The two letters have been discovered by a famous biographer of Kepler—**Max Caspar, who worked in 1935 in the archive of Kepler,** kept in the Pulkovo Observatory, near S. Peterburg, Russia (Kepler's manuscripts were bought by order of the Empress of Russia Екатерина II Великая (Catherine the Great) in 1774).
- While searching through a copy of Kepler's Rudolphine Tables, Caspar found a slip of paper, that had seemingly been used as a book mark.
- It was this slip of paper that contained **Schickard's original drawings of the machine** (from the second letter to Kepler). Later Max Caspar stumbled upon the other pages of the two letters.

Instruction to the Mechanician

- In 1950s another biographer of **Kepler—Dr. Franz Hammer** (1898-1969), made a connection between the two letters from Pulkovo and a sketch of a machine (along with instructions to the mechanic (probably to **Johann Pfister**))
- It described in Schickard's manuscripts (Schickard sketch book), kept in Württembergischen Landesbibliothek in Stuttgart .



Who was the first?

- Caspar and Hammer however were not the first men, who noticed the machine of Schickard.
In 1718 one of the first biographers of Kepler—the German **Michael Gottlieb Hansch** (1683-1749), published a book of letters of Kepler, which includes the two letters from Schickard to Kepler. There is even a marginal note of the publisher *Schickardi machina arithmetica* at the second letter, obviously on the calculating machine.
- In 1899 in the Stuttgart's surveying magazine *Stuttgarter Zeitschrift für Vermessungswesen* was published an old article for the topography in Württemberg, Germany, written many years ago and probably published in other editions, by the famous German scientist **Johann Gottlieb Friedrich von Bohnenberger (1765–1831)**.
- In this article the name of Schickard is mentioned several times, not only concerning his important contribution in the field of topography, but it is mentioned also that *“...it is strange, that nobody admitted, that Schickard invented a calculating machine. In 1624 he ordered a copy for Kepler, but it was destroyed in a night fire.”*
- Bohnenberger (known mainly as the inventor of the gyroscope effect), just like Schickard, studied and later was appointed a professor of mathematics and astronomy at the University of Tübingen since 1798.

100 Years before

- In 1912 in the yearly German magazine *Nachrichten des Württembergischen Vermessungstechnischen Vereins* was published the sketch and the notes of the machine from the *Württembergischen Landesbibliothek*.
- The author of the article A. Georgi was however probably not aware of the two letters of Schickard, but only with the note of Bohnenberger.
- He even claimed, that Leibniz was aware of the machine of Schickard and accused him of plagiarism, which is unbelievable.

Schickard – the first inventor of calculating machine

- In April 1957, Hammer announced his discovery during the conference about the history of mathematics in Oberwolfach, Germany.
- From this moment on, gradually it was made known to the general public, that namely Schickard, but not Blaise Pascal, is the inventor of the first mechanical calculating machine.
- In 1960 Mr. Bruno v. Freytag Löringhoff, professor of philosophy at the University of Tübingen, created the first replica of the Schickard's machine.

Replica from 1960



Wilhelm Schickard to Kepler in Linz, 20. September, 1623,

- The first letter includes - letters are written in Latin language (translated to English):

I have tried to discover a mechanical way for performing calculations, which you have done manually till now. I constructed a machine, which includes eleven full and six partial pinion-wheels, which can calculate automatically, to add, subtract, multiply and divide. You would rest satisfied, if you can see how the machine accumulates and shifts to the left tens and hundreds, and makes the opposite shift during a subtraction....

Tabulae Rudolphinae

- **From 1612 to 1626, Kepler lived in Linz, Austria**, where he worked as a mathematics teacher and as an astrologer.
- In this period (1623), he was completing his famous *Tabulae Rudolphinae* and certainly needed such calculating instrument.
- He must have written back asking for a copy of the machine for himself, because the second letter, dated February 25th, 1624, includes description of the machine with two drawings and bad news about a fire, which destroyed the machine:

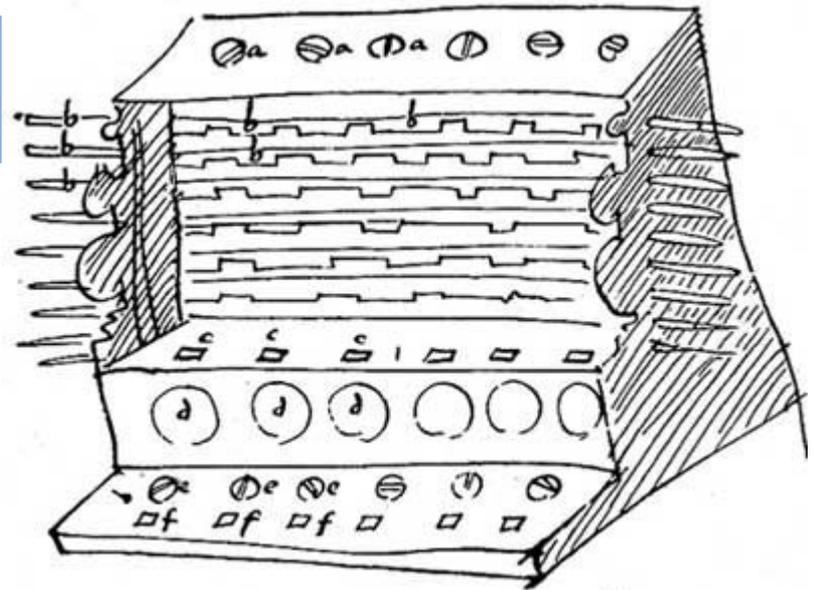
Second letter

...I will describe the computer more precisely some other time, now I don't have enough time:

aaa are the upper faces of vertical cylinders (see the upper figure), whose side surfaces are inscribed with multiplication tables.

*The digits of these tables can be looked out of the windows **bbb** of a sliding plate.*

*From the inner side of the machine to the disks **ddd** are attached wheels with 10 cogs, and each wheel is clutched with a similar wheel in a manner that, provided some of the right wheels spins round ten revolutions, the left wheel will make one revolution, or provided the first wheel spins round 100 revolutions, the third wheel to the left will make one revolution.*

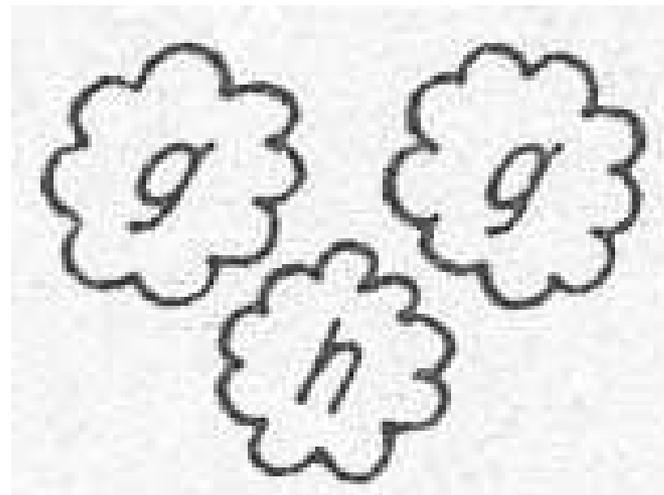


*In order the revolutions of the wheels to be in the same direction, intermediate wheels **h** are necessary.*

Each intermediate wheel moves to the left needed carry, but not to the right, which made special caution measures necessary.

Second letter 2

- *The digits, inscribed upon the each wheel, can be looked out of the windows **ccc** of the middle bank. In the end of the lower bank are arranged rotating heads **eee**, used for recording of numbers, which are the result of the calculations, and their digits can be looked out of the windows **fff**.*
- *I have already ordered a copy for you to our Johann Pfister, together with some other things for me, especially some copper plates, but when the work was half finished, yesterday night a fire burst out and everything burnt out, as Maestlin informed you. I take this loss very heavily, because there is no time for its replacement.*



Schickard obviously was not satisfied of the work of the mechanic, involved in the production of the device, because the note to him begins:

*Concerning Calculating Clock,
1. The teeth are inequally made and don't work...*

Machine for Kepler?

- That's the whole information, survived up to the present for **the *Calculating Clock* of Schickard**.
- It seems the prototype of the machine, mentioned in the first letter, was rather successful, that's why Schickard ordered the next copy for Kepler.
- It is unknown whether another copy was ever created, and how many devices are made or ordered by the inventor. It is out of the question however, that such device has not been delivered to Kepler.
- Most probably, only two machines were produced, the prototype, mentioned in the first letter, which was in the home of Schickard, and disappeared after his death, and **second, made for Kepler, which was destroyed during the fire**.

The structure and function of device

The structure designed by Schickard and produced by Pfister:

The *Calculating Clock* is composed of 3 main parts:

- A multiplying device.
- A mechanism for recording of intermediate results.
- A decimal 6-digits adding device.

The multiplying device

is composed of 6 vertical cylinders with inscribed numbers of Napier's rods.



Multiplying Device

- From the front side the cylinders are covered with **9 narrow plates with windows**, which can be moved leftwards and rightwards.
- After entering of the multiplicand by rotating of the cylinders through the knobs in upper side of the box, by means of opening of the windows of plates can be made consecutive multiplying first by units of the multiplier, then by tens and so on.
- **The intermediate products can be added by means of adding device.**

Intermediate Results

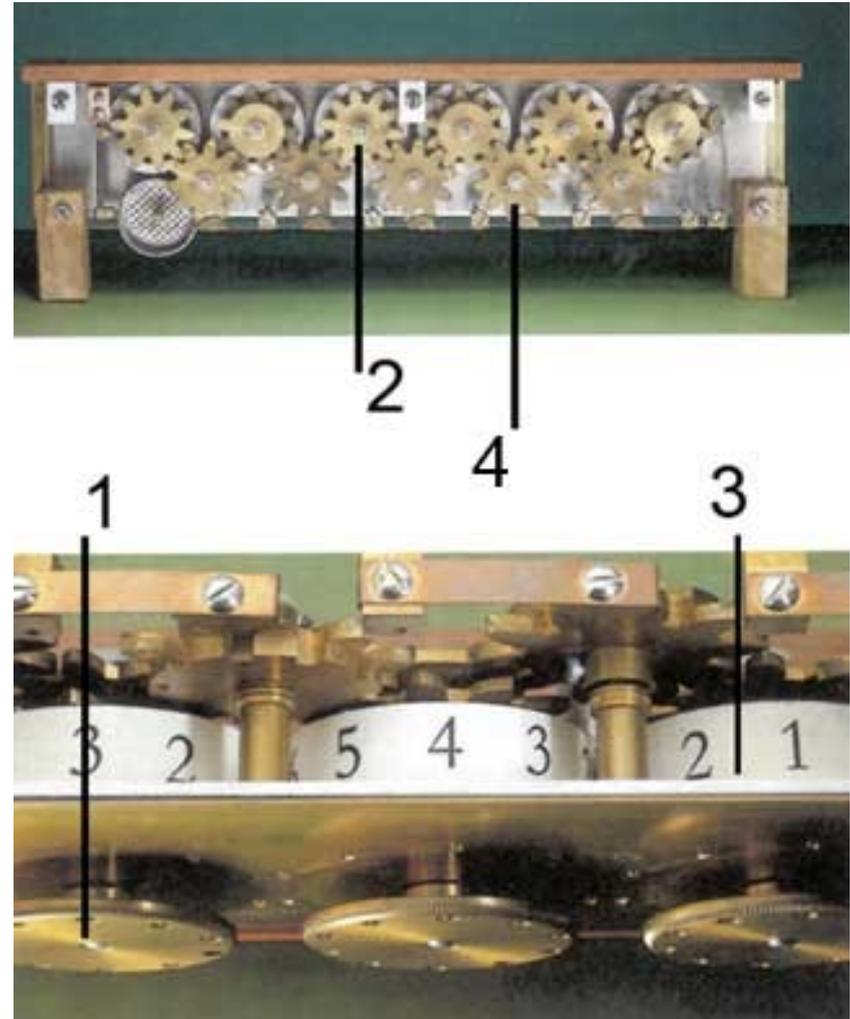
- The mechanism for recording of intermediate results of calculations is composed by 6 rotating through small knobs disks with peripheries inscribed with digits, which can be seen in the small windows in the lower row.
- These disks are not connected with the calculating mechanism and don't have a tens carry mechanism.



The Adding Device

The adding device is composed of six basic axes in a row.

- On each axis is mounted a smooth disk with ten openings (marked with 1 in the lower photo), a cylinder with inscribed digits (marked with 3), and a pinion-wheel with 10 teeth (marked with 2), over which is fixed pinion-wheel with 1 tooth (which are used for tens carry).
- On other 5 axes are mounted pinion-wheels with 10 teeth (marked with 4).



Entering of Numbers

- The smooth disks are used for entering of the numbers and resetting of the machine.
- The digits on the inscribed cylinders can be seen in the upper row of windows and are used for reading of the results of adding and subtracting operations.
- Over the each of the 10-teeth disks on the basic axes is mounted a one-tooth disk, in such manner, that for each full revolution of 10-teeth disk, 1-tooth disk enters once in a contact with the according intermediate disk and rotates it to $1/10$ revolution.
- This is the mechanism of tens carry. The axes can be rotated in both directions, so the machine can be used not only for addition, but for direct subtraction too (no need to use the arithmetical operation *complement to 9*, as it was the case with Pascaline).
- Due to the intermediate disks, all smooth disks are rotated in the same direction.
- The machine has also a indicator for overflow—a small bell, which rings if the leftmost pinion-wheel rotates from 9 to 0.

Example

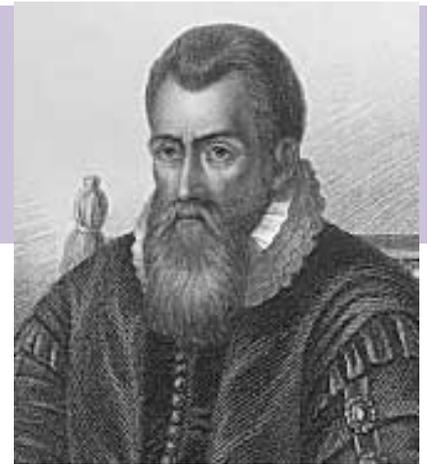
- Lets make a simple multiplication with the machine, for example 524×48 .
- First we have to rotate the rightmost cylinder to 4, next cylinder to 2, and the third from right to 5 (the multiplicand is 524).
- Then we have to open the windows on the 8th row (units of the multiplier are 8) and we will see in the windows the first intermediate result (4192).
- We have to enter the 4192 in the calculating mechanism.
- Then we have to open the windows on the 4th (tens of multiplier are 4) row and to see the second intermediate result—20960, which we have to enter to the calculating mechanism, and we will have the result—25152.

Two faults

1. The inventor didn't describe a means for fixing of the intermediate disks, which is certainly necessary. As you can see in the photos, the technicians of Mr. Freytag Löringhoff have provided such mechanism (the small disks bellow the intermediate disks).
2. The second problem is the friction. In the beginning of the 17th century the turret lathes had not been invented yet, so the pinion-wheels have to be produced manually and with great precision, otherwise the friction in case of full carrying (for example when to 999999 must be added 1) will be enormous and the machine will be hard for operating and easy to broken.

Schickard obviously had faced such problems, and that's why his machine has only six main axes, in spite of the vital necessity of Kepler to work with big numbers for his astronomical calculations.

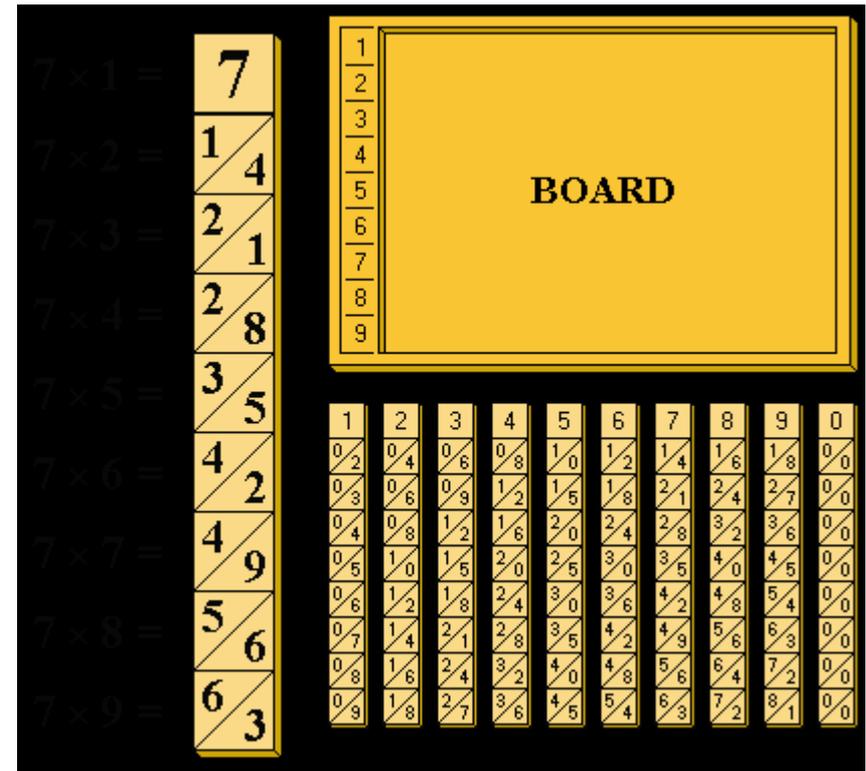
Napier bones



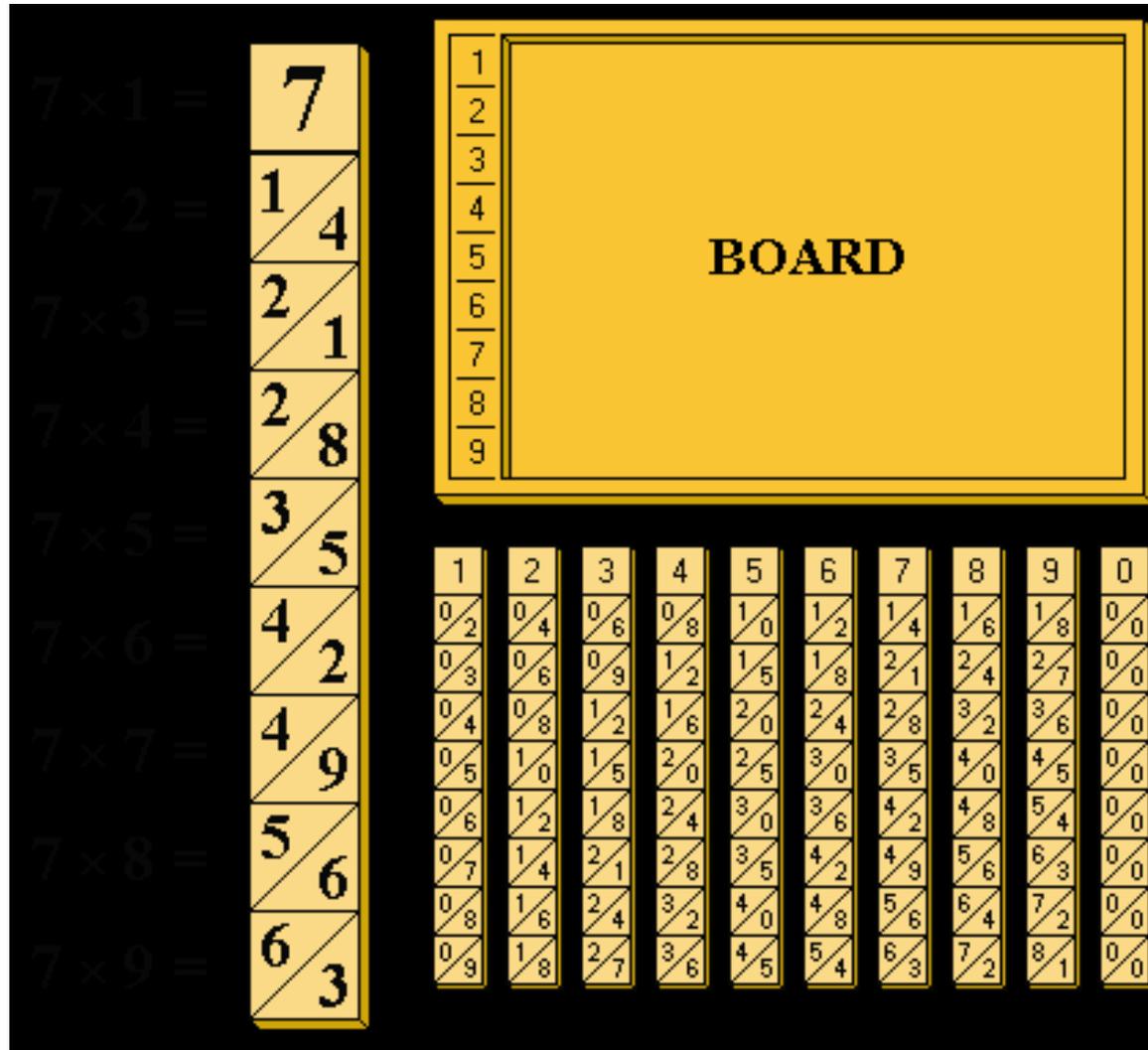
- **Napier's bones** is an **abacus** created by John Napier for calculation of products and quotients of numbers that was based on Arab mathematics and lattice multiplication used by **Fibonacci writing in the Liber Abaci**.
- Also called **Rabdology** (from Greek $\rho\acute{\alpha}\beta\delta\omicron\varsigma$ [r(h)abdos], "rod" and $-\lambda\omicron\gamma\acute{\iota}\alpha$ [logia], "study").
- Napier published his version of rods in a work printed in Edinburgh in Scotland, at the end of 1617 entitled *Rabdologiæ*.
- Using the multiplication tables embedded in the rods, multiplication can be reduced to addition operations and division to subtractions.
- More advanced use of the rods can even extract square roots.
- Note that **Napier's bones** are not the same as logarithms, with which Napier's name is also associated.
- The abacus consists of a board with a rim; the user places Napier's rods in the rim to conduct multiplication or division.
- The board's left edge is divided into 9 squares, holding the numbers 1 to 9.

Napier rods or Napier bones

- The **Napier's rods** consist of strips of wood, metal or heavy cardboard.
- **Napier's bones** are three dimensional, square in cross section, with four different **rods** engraved on each one. A set of such **bones** might be enclosed in a convenient carrying case.
- A rod's surface comprises 9 squares, and each square, except for the top one, comprises two halves divided by a diagonal line. The first square of each rod holds a single-digit, and the other squares hold this number's double, triple, quadruple and so on until the last square contains nine times the number in the top square.
- The digits of each product are written one to each side of the diagonal; numbers less than 10 occupy the lower triangle, with a zero in the top half.
- A set consists of 10 rods corresponding to digits 0 to 9.
- The rod 0, although it may look unnecessary, is obviously still needed for multipliers or multiplicands having 0 in them.

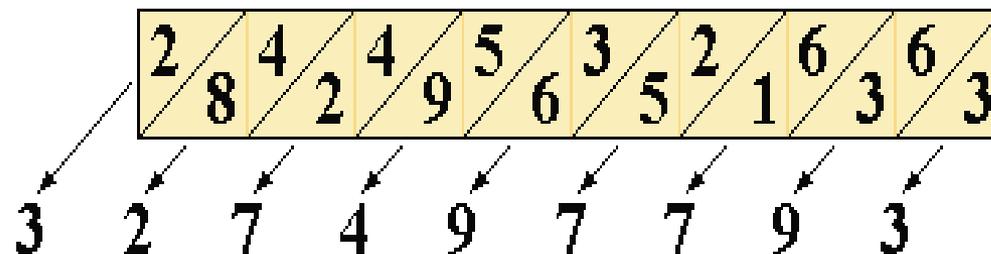


Napier bones



Example of Multiplication

1	4	6	7	8	5	3	9	9	
2	0/8	1/2	1/4	1/6	1/0	0/6	1/8	1/8	
3	1/2	1/8	2/1	2/4	1/5	0/9	2/7	2/7	
4	1/6	2/4	2/8	3/2	2/0	1/2	3/6	3/6	
5	2/0	3/0	3/5	4/0	2/5	1/5	4/5	4/5	
6	2/4	3/6	4/2	4/8	3/0	1/8	5/4	5/4	
7	2/8	4/2	4/9	5/6	3/5	2/1	6/3	6/3	
8	3/2	4/8	5/6	6/4	4/0	2/4	7/2	7/2	
9	3/6	5/4	6/3	7/2	4/5	2/7	8/1	8/1	



$$46\,785\,399 \times 7 = 327\,497\,793$$

The Pascaline of Blaise Pascal

- The *Pascaline* of Blaise Pascal was for long time considered as the first mechanical calculator in the world.



It is more likely Pascal to have read the *Annus Positionum Mathematicarum* of Dutch Jesuit-mathematician *Jan Ciermans* (1602-1648), who mentioned in this book, that he created a mechanical calculator with *iron wheels*, suitable for multiplication and division.

How it all began?

- In 1639 **Étienne Pascal**, father of the great french scientist Blaise Pascal, was appointed by the **Cardinal de Richelieu as *Commissaire député par sa Majesté en la Haute Normandie*** (financial assistant to the intendant Claude de Paris) in Rouen, capital of the Normandy province.
- **Étienne Pascal** arrived in the city of Rouen in January 1640. He was a meticulous, forthright and honest man, and spent a **considerable amount of his time completing arithmetic calculations for taxes.**
- The task of calculating enormous amounts of numbers in millions of deniers, sols and livres necessitated ultimately the help of his son Blaise and one of his cousins'son, Florin Perrier, who would soon marry Blaise's sister Gilberte.
- **The young men used initially only manual calculations and abacus, but in 1642 the Blaise started to design a calculating machine.**
- The first variant of the machine was ready next year, and the young genius continue his work on improving of his calculating machine.

First devices

- First several copies of the machine didn't satisfied the inventor.
- Then happened an event, which almost manage to give up Pascal from the machine. A watchmaker from Rouen *dared*, (according to the words of the offended inventor, who named no name—whether he knew it is unknown), to “*make a beautiful, but absolutely useless for work copy of my machine... The appearance of this monster was so painful for me and so damped my enthusiasm to work on the new model, that I settled accounts with my mechanics and decide to give up my idea*”.

Royal Privilege

- Later on, however, friends of Pascal presented to the Chancellor of France, Pierre Segulier (1588–1672), a prototype of the calculating machine. Segulier admired the invention and encouraged Pascal to resume the development.
- In 1645 Pascal wrote a dedicatory letter at the beginning of his **pamphlet** (*Advis Nécessaire*) **describing the machine**, and donated a copy of the machine to the Chancellor (still preserved in CNAM, Paris).
- The text concluded that the machine could be seen in operation and purchased at the residence of Roberval. This is the only preserved description of the device from the inventor.
- The Chancellor Segulier continued to support Pascal and in May, 1649, by royal decree, signed by Louis XIV of France, **Pascal received a monopoly (privilege) on the arithmetical machine**, according to which *the main invention and movement is this, that every wheel and axis, moving to the 10 digits, will force the next to move to 1 digit and it is prohibited to make copies not only of the machine of Pascal, but also of any other calculating machine, without permission of Pascal. It is prohibited for foreigners to sell such machines in France, even if they are manufactured abroad. The violators of the privilege will have to pay penalty of 3 thousand livres* (

Next prototypes?

- The privilege mentions that Pascal has already produced fifty somewhat different prototypes. Moreover, the patent was awarded gratis and did not specify an expiration time, which was rather unusual affair. It seems Pascal was an authentic favorite of the French crown :-)
- It seems later Pascal wanted to manufacture his machines as a full scale business enterprise, but it proved too costly, and he didn't manage to make money from this privilege.
- Price may have been the main issue here, though accounts vary significantly, from the Jesuit mathematician François's 100 livres to Tallemant de Réaux's 400 livres and Balthasar Gerbier's 500 livres.

Famous Pascaline

- The *Pascaline* soon become well-known in France and abroad.
- The first public description was in 1652, in the newspaper *Muse Historique*.
- The machine was demonstrated to the public in Paris.
- The Polish queen **Marie Louise de Gonzague**, a high-ranking and keen patron of sciences, asked to buy two copies.
- Another fan of sciences, **Swedish queen Christina** desired a copy to be granted to her.
- Pascal satisfied her desire, but soon after this lost his interest and abandoned his occasions with the calculation machine until the end of his short life.

Pascaline from 1652



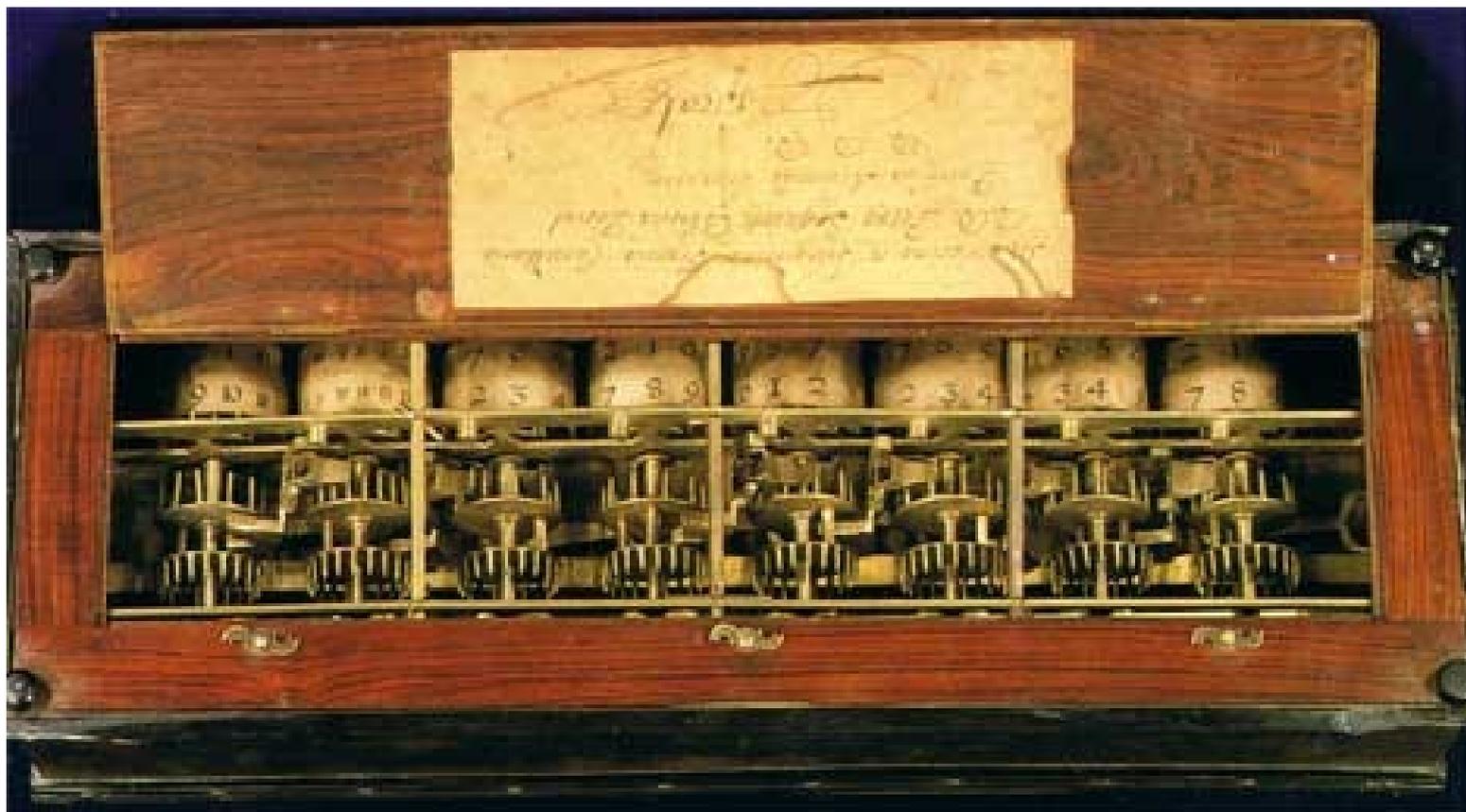
8-9 copies to present day

- From some 50 constructed Pascalines, only 8-9 survived to the present day, and can be seen in private or museum collections (4 in CNAM, Paris, 2 in a museum in Clermont, and several in private collections, e.g. in IBM).
- First copies of the machine were with **five digital positions**.
- Later on Pascal manufactured machines **with 6, 8, and even 10 digital positions**.
- Some of the machines are entirely decimal (i.e. the scales are divided to 10 parts), others are destined to monetary calculations and have scales with 12 and 20 parts (according to **French monetary units: 1 sol = 12 deniers, 1 livre = 20 sols**).

Description of Pascaline

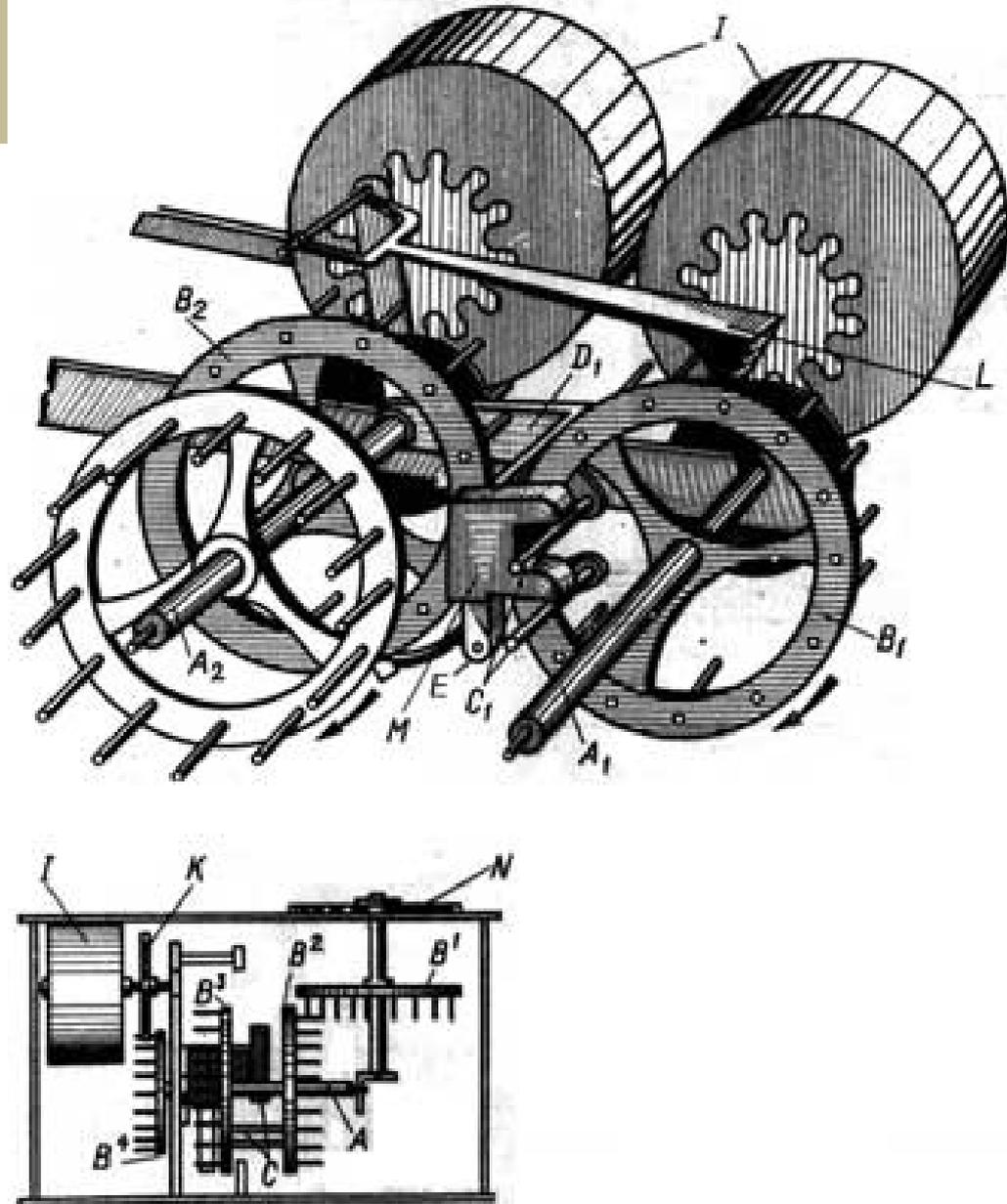
- The dimensions of brass box of the machine (for 8 digital positions variant) is 35.1/12.8/8.8 cm.
- The input wheels are divided by 10, 12 or 20 spokes, depending of the scale. The spokes are used for rotating of the wheels by means of a pin or stylus.
- The stylus rotates the wheel until it get to an unmovable stop, fixed to the lower part of the lid.
- The result can be seen in the row of windows in upper part, where is placed a plate, which can be moved upwards and downwards, allowing to be seen upper or lower row of digits, used for addition or subtraction.

Open Box



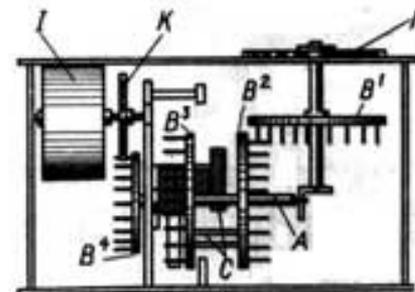
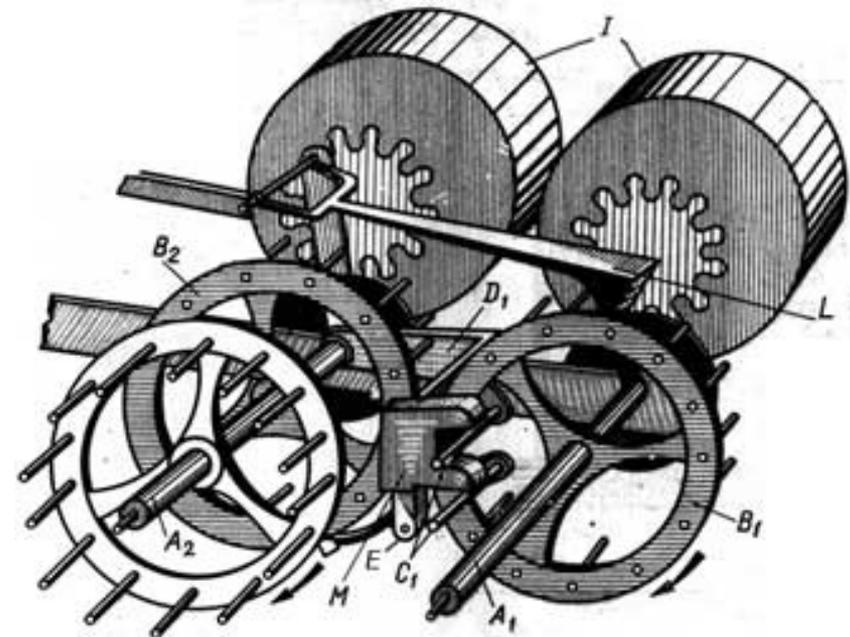
Sketch of Pascaline

- The input wheels (used for entering of numbers) are smooth wheels, across which periphery are made openings.
- Counter-wheels are crown-wheels, i.e. they have openings with attached pins across periphery.
- The movement is transferred from the input wheel (marked with N in the sketch), which can be rotated by the operator by means of a stylus, over the counter, which consists of four crown-wheels (marked with $B1$, $B2$, $B3$ and $B4$), pinion-wheel (K), and mechanism for tens carry (C), to the digital drum (I), which digits can be seen in the windows of the lid.



How it work?

- The tens carry mechanism (called by Pascal *sautoir*), works in this way:
- On the counter-wheel of the junior digital positions (**B1**) are mounted two pins (**C1**), which during the rotating of the wheel around its axis (**A1**) will engaged the teeth of the fork (**M**), placed on the edge of the 2-legs rod (**D1**).
- This rod can be rotated around the axis (**A2**) of the senior digital position, and fork has a tongue (**E**) with a spring.
- When during the rotating of the axis (**A1**) the wheel (**B1**) reach the position, according to the digit 6, then pins (**C1**) will engaged with the teeth of the fork, and in the moment, when the wheel moves from 9 to 0, then the fork will slide off from the engagement and will drop down, pushing the tongue. It will push the counter wheel (**B2**) of the senior position one step forward (i.e. will rotate it together with the axis (**A2**) to the appropriate angle.
- The rod (**L**), which has a special tooth, will serve as a stop, and will prevent the rotating of the wheel (**B1**) during the raising of the fork.
- The tens carry mechanism of Pascal has an advantage, compared i.e. to this of Schickard's Calculating Clock, because it is needed only a small force for transferring the motion between adjacent wheels.
- This advantage however is paid by a some shortcomings—during the carrying is produced a noise, and if the box is hit, may occur unwanted carrying.

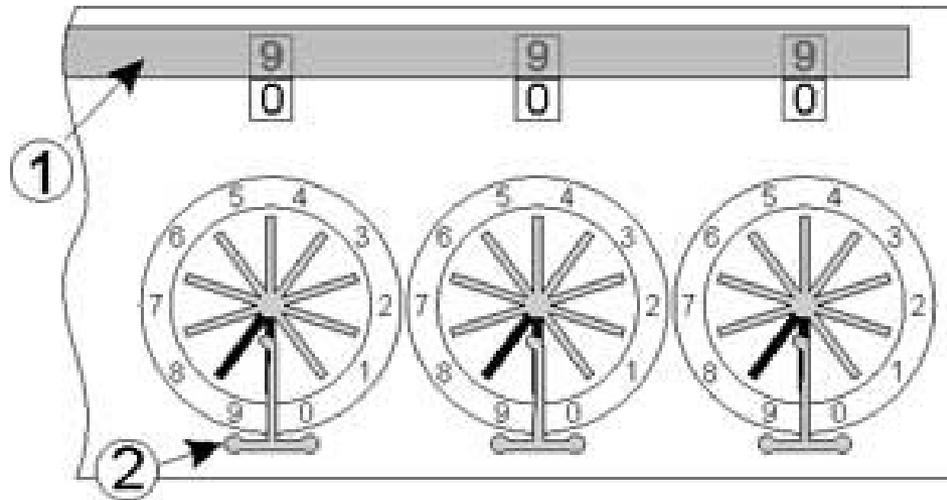


Device is only for Adding

- The wheels of the calculating mechanism are rotating **only in one direction**.
- This means, that the machine can work only as a adding device, and subtraction must be done by means of a arithmetical operation
(known as complement to 9).
- This inconvenience can be avoided by adding of additional intermediate gear-wheels in the mechanism, but Pascal, as well as all next inventors of calculating machines (Leibniz, Lepine, Leupold, etc.) didn't want to complicate the mechanism and didn't provide such possibility.

Zeroing of the mechanism of Pascaline

- Zeroing of the mechanism can be done by rotating of the wheels by means of the stylus in such way, that between the two starting spokes (marked on the wheel) to be seen 9.
- In this moment the digits of the lower row will be 0, while the upper digits will be 9 (or 12 or 20, for sols and deniers).



- The work with the Pascaline is not very easy, but the machine is completely usable for simple calculations.