An aerial photograph of a city skyline, likely Chicago, featuring numerous skyscrapers and a large body of water (Lake Michigan) in the background under a cloudy sky. The text is overlaid on the center of the image.

Mathematics in the 20th and 21st Century

Alena Šolcová,
FIT CTU in Prague

David Hilbert

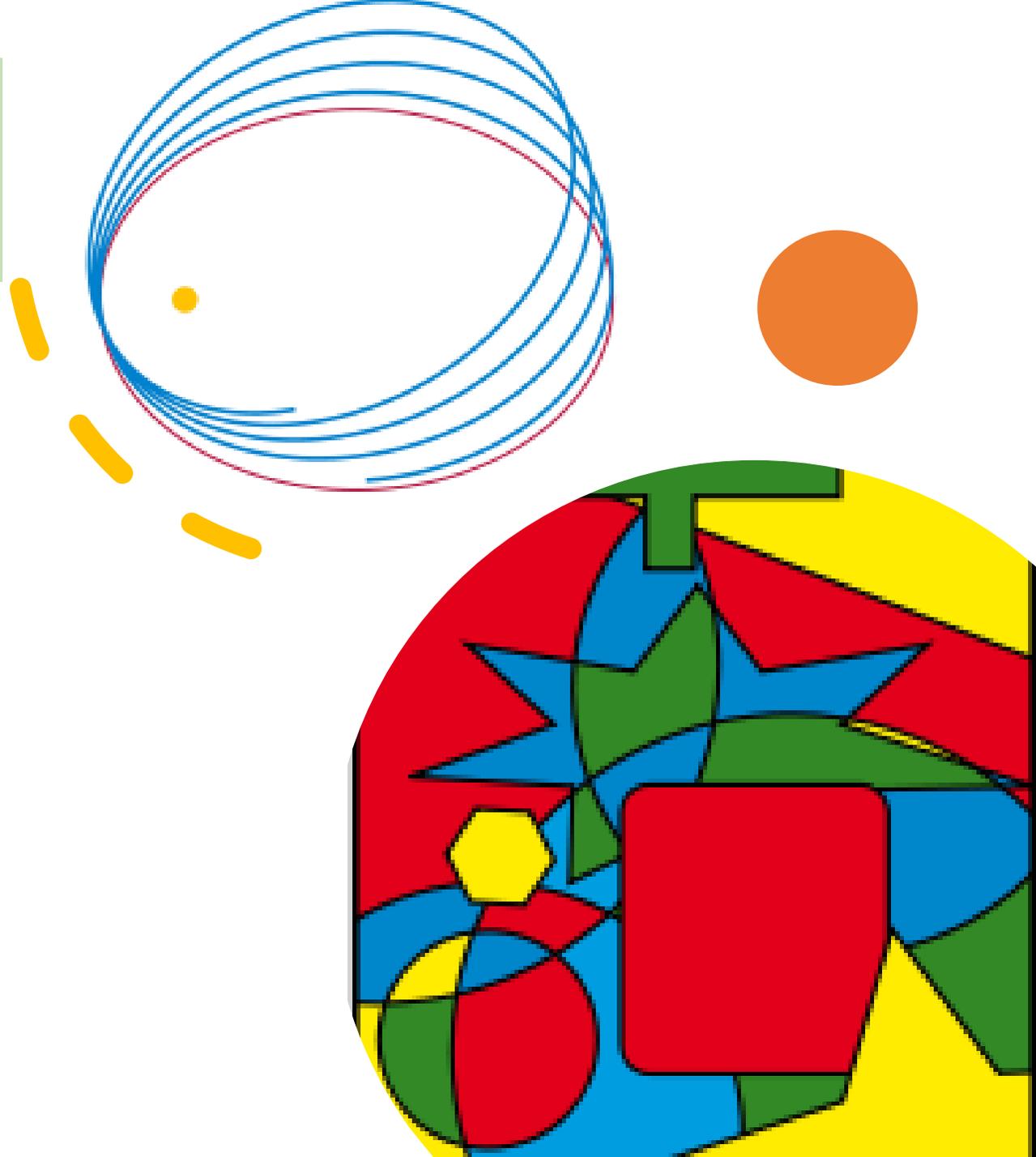
23 Unsolved Problems

In a **1900** speech to the International Congress of Mathematicians, David Hilbert set out **a list of 23 unsolved problems in mathematics.**

These problems, spanning many areas of mathematics, formed a central focus for much of 20th-century mathematics.

Today, **10 have been solved**, 7 are partially solved, and 2 are still open.

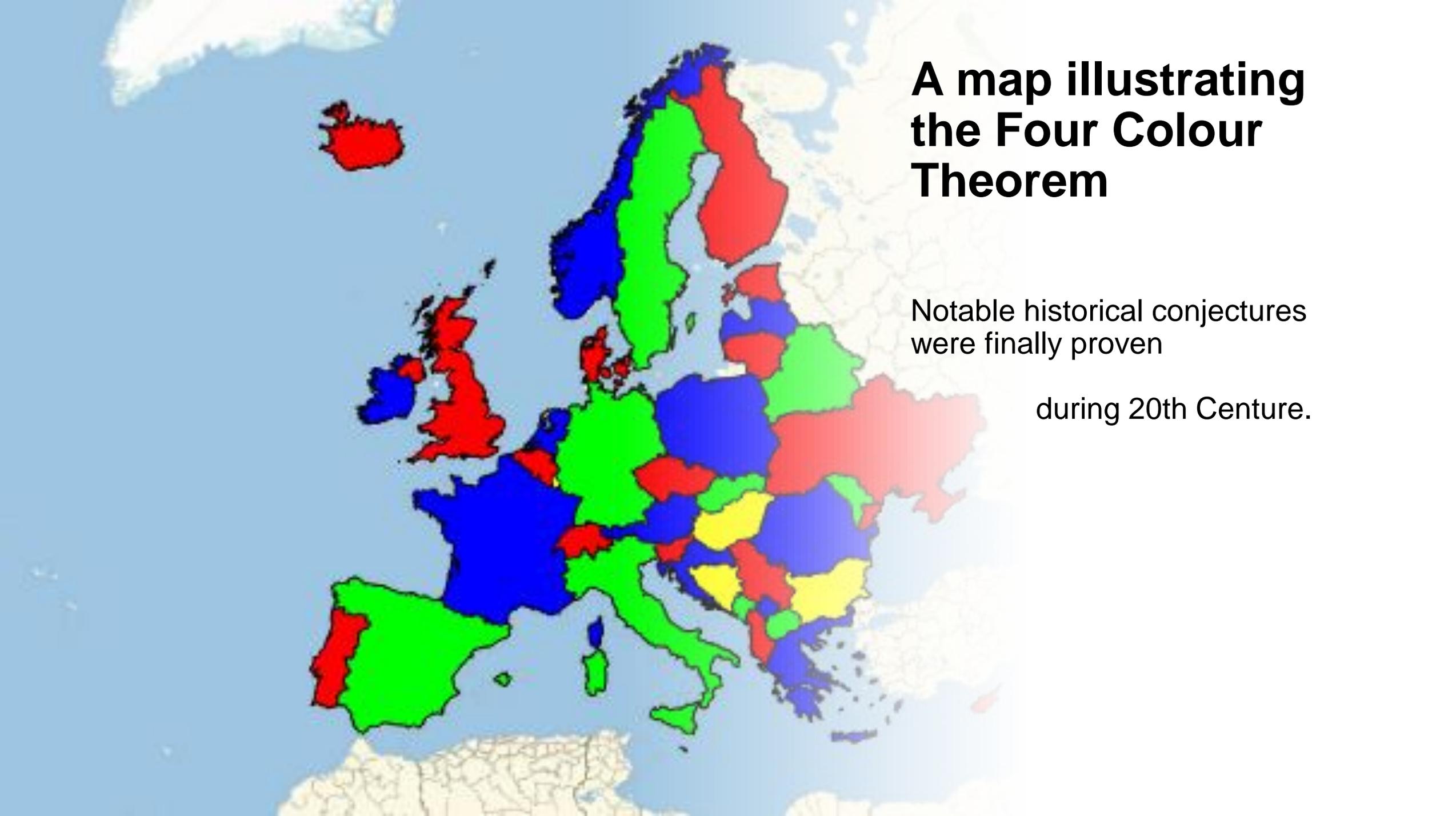
The remaining 4 are too loosely formulated to be stated as solved or not.

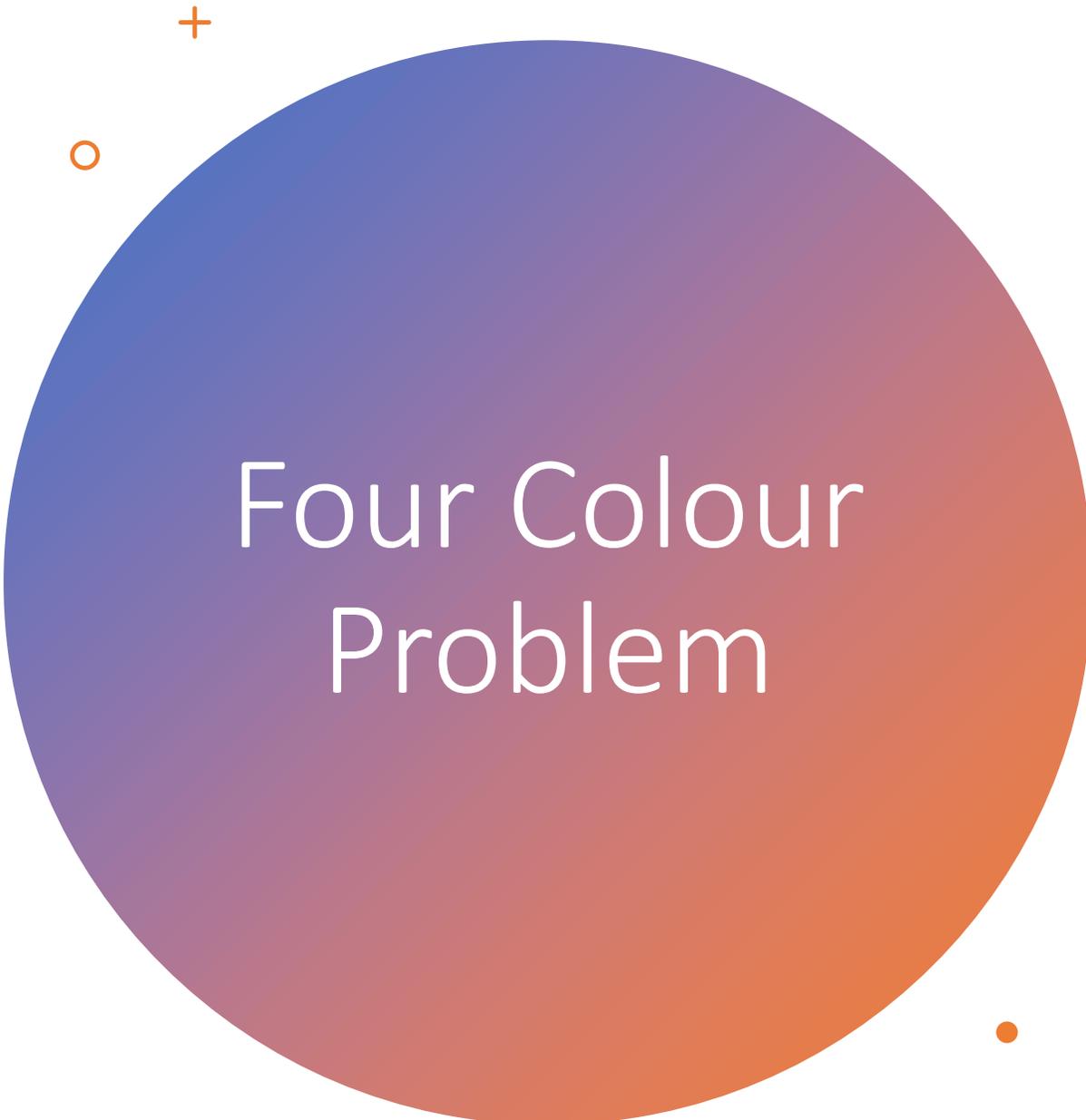


A map illustrating the Four Colour Theorem

Notable historical conjectures
were finally proven

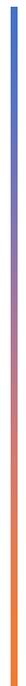
during 20th Centure.



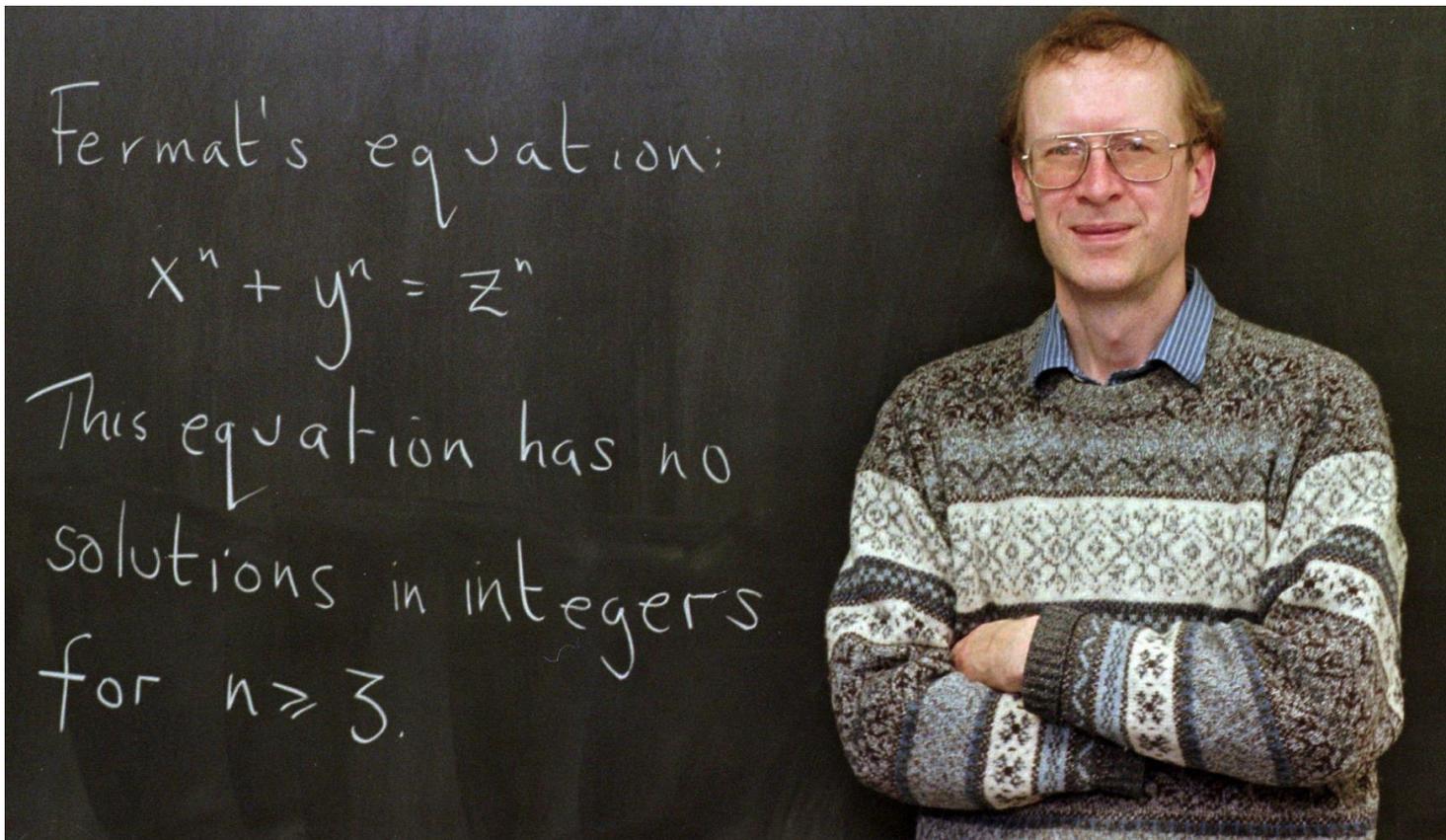


Four Colour Problem

**In 1976,
Wolfgang Haken
and
Kenneth Appel
proved
the four colour theorem,
controversial
at the time
for the use
of a computer to do so.**



Fermat's Last Theorem



Andrew Wiles,
building on the
work of others,

proved
**Fermat's Last
Theorem**

in 1995.

Mathematical collaboration

Famous
Nickname

—

Nicolas
Bourbaki

Paul Cohen and Kurt Gödel proved that the continuum hypothesis is independent of (could neither be proved nor disproved from) the standard axioms of set theory.

In 1998 Thomas Callister Hales proved the Kepler conjecture.

Mathematical collaborations of unprecedented size and scope took place.

An example is the classification of finite simple groups (also called the "enormous theorem"), whose proof between 1955 and 2004 required 500-odd journal articles by about 100 authors, and filling tens of thousands of pages.

A group of French mathematicians, including Jean Dieudonné and André Weil, publishing under the pseudonym "Nicolas Bourbaki", attempted to exposit all of known Mathematics as a coherent rigorous whole.

The resulting several dozen volumes has had a controversial influence on mathematical education.

Alan Turing's
Computability
Theory

Derrick Lehmer
use
of ENIAC

Alan Turing's **computability theory**
complexity theory

Derrick Henry Lehmer's **use of ENIAC**

to further number theory and
the Lucas-Lehmer test.



Significant Personalities

—

RAMANUJAN,
Paul ERDÖS,
Emmy
NOETHER

One of the more colorful figures in 20th-century mathematics was **Srinivasa Aiyangar Ramanujan** (1887–1920), an Indian autodidact who conjectured or proved

over 3000 theorems, including **properties of highly composite numbers,** the **partition function** and its asymptotics, and **mock theta functions.**

He also made major investigations in the areas of **gamma functions,** **modular forms,** **divergent series,** **hypergeometric series** and **prime number theory.**

Paul Erdős (1913 – 1996)

Paul Erdős published more papers than any other mathematician in history, working **with hundreds of collaborators.**

Mathematicians have a game equivalent to the **Kevin Bacon Game**, which leads to **the Erdős number** of a mathematician.

This describes the „**collaborative distance**“ between person and Paul Erdős, as measured by joint authorship of mathematical papers.



Emmy
Noether
(1882 -1935)

Emmy Noether
has been
described by many
**as the most
important woman**
in the history
of mathematics.

She studied
**the theories
of rings,
fields,
and algebras.**



New Areas of Mathematics, e.g. Differential Geometry or Game Theory

Differential geometry came into its own when Albert Einstein used it in general relativity.

Entirely new areas of mathematics such as **mathematical logic, topology,** and **John von Neumann's game theory** changed the kinds of questions that could be answered by mathematical methods.

All kinds of structures were abstracted using axioms and given names like **metric spaces, topological spaces** etc. As mathematicians do, the concept of an abstract structure was itself abstracted and led to **category theory**.

Grothendieck and Serre recast **algebraic geometry** using **sheaf theory**.

New Areas of
Mathematics,
e.g. Dynamical
Systems, etc.

- Large advances were made in the qualitative study of **dynamical systems** that **Poincaré** had begun in the 1890s.

Measure theory was developed in the late 19th and early 20th centuries. Applications of measures include **the Lebesgue integral, Kolmogorov's axiomatisation of probability theory,** and **ergodic theory.**

Knot theory greatly expanded. Quantum mechanics led to the development of **functional analysis.**

Other new areas include **Laurent Schwartz's distribution theory, fixed point theory, singularity theory** and **René Thom's catastrophe theory, model theory,** and **Mandelbrot's fractals.** **Lie theory** with its **Lie groups** and **Lie algebras** became one of the major areas of study.

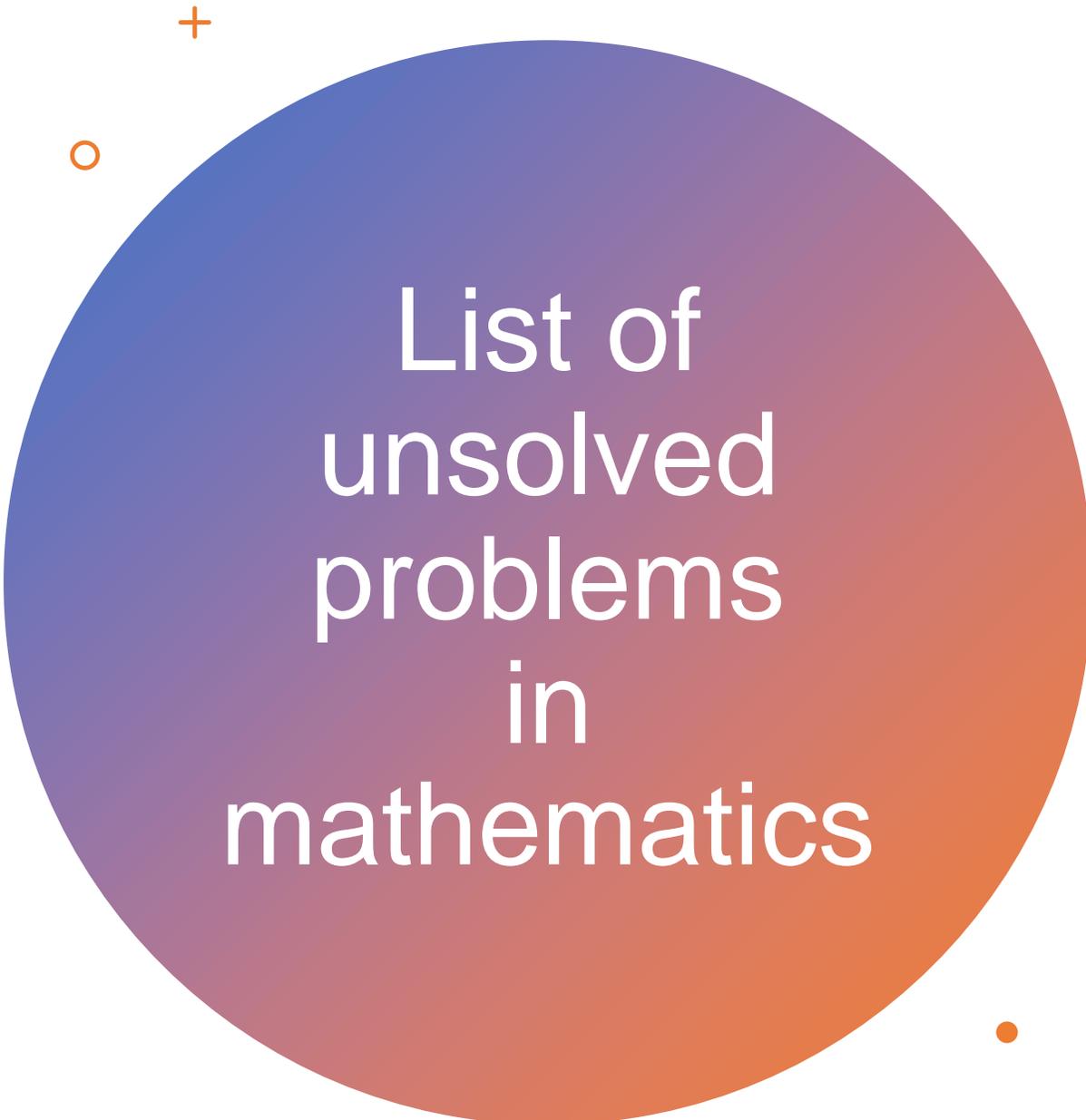
The End of 20th Century

As in most areas of study,
the explosion of knowledge
in the scientific age **has led to specialization.**

By the end of the century there were
hundreds of specialized areas in mathematics
and **the Mathematics Subject Classification**
was dozens of pages long.

More and more **mathematical journals**
were published.

By the end of the century,
the development of **the World Wide Web**
led to online publishing.



List of
unsolved
problems
in
mathematics

Problems solved since 1995.

In 2000, the **Clay Mathematics Institute** announced the **seven Millennium Prize Problems**,

In 2003 the **Poincaré conjecture** was solved by **Grigori Perelman** (who declined to accept an award, as he was critical of the mathematics establishment).

Most mathematical journals now have online versions as well as print versions, and many online-only journals are launched.

There is an increasing drive toward **open access publishing**,

+

•

○

Future of mathematics

There are many observable trends in mathematics, the most notable being that **the subject is growing ever larger, computers are ever more important and powerful.**

The application of mathematics to bioinformatics is rapidly expanding, and the volume of data being produced by science and industry, facilitated by computers, is explosively expanding.